(7)

$$\alpha+\beta+\gamma=-p \quad \text{——} (1)$$
$$\alpha\beta+\beta\gamma+\gamma\alpha=q \quad \text{——} (2)$$

ow $\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$= p^2-2q$.

$$\alpha+\beta+\gamma=-\frac{b}{a} \quad \text{——} (1)$$
$$\alpha\beta+\beta\gamma+\gamma\alpha=\frac{c}{a} \quad \text{——} (2)$$
$$\alpha\beta\gamma=-\frac{d}{a} \quad \text{——} (3)$$

ow $\alpha^3+\beta^3+\gamma^3-3\alpha\beta\gamma=(\alpha+\beta+\gamma)(\alpha^2+\beta^2+\gamma^2-\alpha\beta-\beta\gamma-\gamma\alpha)$

$\alpha^3+\beta^3+\gamma^3+3\frac{d}{a}=-\frac{b}{a}\left\{(\alpha+\beta+\gamma)^2-3(\alpha\beta+\beta\gamma+\gamma\alpha)\right\}$

$\Rightarrow \alpha^3+\beta^3+\gamma^3=\frac{3b}{a}\cdot\frac{c}{a}-\frac{b}{a}\left(-\frac{b}{a}\right)^2-3\frac{d}{a}$

$=\frac{3bc}{a^2}-\frac{b^3}{a^3}-\frac{3d}{a}=\frac{3abc-b^3-3a^2d}{a^3}$

$$\Sigma\alpha=-\frac{b}{a}, \quad \text{——} (1)$$
$$\Sigma\alpha\beta=\frac{c}{a} \quad \text{——} (2)$$
$$\alpha\beta\gamma=-\frac{d}{a} \quad \text{——} (3)$$

Now $\Sigma\alpha^2\beta=\alpha^2\beta+\beta^2\gamma+\gamma^2\alpha+\alpha^2\gamma+\beta^2\alpha+\gamma^2\beta$

$=\cancel{\alpha^2(\beta+\gamma)+\beta^2(\gamma+\alpha)+\gamma^2(\alpha+\beta)}$

$=\alpha^2\beta+\beta^2\alpha+\alpha\beta\gamma+\alpha\beta\gamma+\beta^2\gamma+\gamma^2\beta+\alpha^2\gamma+\alpha\beta\gamma+\gamma^2\alpha-3\alpha\beta\gamma$

$=\alpha\beta(\alpha+\beta+\gamma)+\beta\gamma(\alpha+\beta+\gamma)-3\alpha\beta\gamma+\gamma\alpha(\alpha+\beta+\gamma)$

$=(\alpha+\beta+\gamma)(\alpha\beta+\beta\gamma+\gamma\alpha)-3\alpha\beta\gamma$

$=-\frac{b}{a}\cdot\frac{c}{a}-3\left(-\frac{d}{a}\right)=\frac{3d}{a}-\frac{bc}{a^2}=\frac{3ad-bc}{a^2}$

For 2nd part see Prob-2.

(4) $\Sigma\alpha = 0$

$\Sigma\alpha\beta = p$

$\alpha\beta\gamma = -q$

(5)

Now $\sum \frac{1}{\alpha+\beta} = \frac{1}{\alpha+\beta} + \frac{1}{\beta+\gamma} + \frac{1}{\gamma+\alpha}$

$= -\frac{1}{\gamma} - \frac{1}{\alpha} - \frac{1}{\beta} = -\left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right)$

$= \frac{-(\alpha\beta + \beta\gamma + \alpha\gamma)}{\alpha\beta\gamma}$

$= \frac{-p}{-q} = \frac{p}{q}$

(5)

$\Sigma\alpha = -p$

$\Sigma\alpha\beta = q$

$\alpha\beta\gamma = -r$

Now $\beta^2\gamma^2 + \gamma^2\alpha^2 + \alpha^2\beta^2$

$= (\beta\gamma + \gamma\alpha + \alpha\beta)^2 - 2\alpha\beta\gamma(\alpha+\beta+\gamma)$

$= q^2 + 2r(-p) = q^2 - 2pr$

(6) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} = \frac{\beta^2\gamma^2 + \gamma^2\alpha^2 + \alpha^2\beta^2}{\alpha^2\beta^2\gamma^2}$

$= \frac{q^2 - 2pr}{r^2}$ Ans.

(6)

$\Sigma\alpha = 0$

$\Sigma\alpha\beta = p$

$\alpha\beta\gamma = -q$

Now $\beta + \gamma - 2\alpha = (\alpha+\beta+\gamma) - 3\alpha = -3\alpha$

$\gamma + \alpha - 2\beta = -3\beta$

$\alpha + \beta - 2\gamma = -3\gamma$

$(\beta+\gamma-2\alpha)(\gamma+\alpha-2\beta)(\alpha+\beta-2\gamma) = (-3\alpha)(-3\beta)(-3\gamma)$

$= -27\alpha\beta\gamma = -27(-q) = 27q$

Let us form the eqn whose roots are

$\beta+\gamma-3\alpha,\ \gamma+\alpha-3\beta,\ \alpha+\beta-3\gamma$

Now $\beta+\gamma-3\alpha = \alpha+\beta+\gamma-4\alpha = -p-4\alpha$

Let $y = -p-4x \Rightarrow x = -\frac{1}{4}(y+p)$

Substituting this in the eqn we get

$-\frac{1}{64}(y+p)^3 + \frac{p}{16}(y+p)^2 - \frac{1}{4}q(y+p) + r = 0$

$(y+p)^3 - 4p(y+p)^2 + 16q(y+p) - 64r = 0$

The Constant term of this eqn $= +p^3 - 4p^3 + 16pq - 64r$

The product of the roots of this eqn

$= (\beta+\gamma-3\alpha)(\gamma+\alpha-3\beta)(\alpha+\beta-3\gamma) = -(+p^3 - 4p^3 + 16pq - 64r)$

$= 3p^3 - 16pq + 64r.$

Let us form the eqn whose roots are

$\frac{1}{\beta}+\frac{1}{\gamma}-\frac{1}{\alpha},\ \frac{1}{\gamma}+\frac{1}{\alpha}-\frac{1}{\beta}$ and $\frac{1}{\alpha}+\frac{1}{\beta}-\frac{1}{\gamma}$.

Now $\frac{1}{\beta}+\frac{1}{\gamma}-\frac{1}{\alpha} = \frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}-\frac{2}{\alpha} = \frac{\alpha\beta+\alpha\gamma+\gamma\beta}{\alpha\beta\gamma} - \frac{2}{\alpha}$

$= -\frac{q}{r} - \frac{2}{\alpha}$

We put $y = -\frac{q}{r} - \frac{2}{x} \Rightarrow x = \frac{-2r}{ry+q}$

eqn takes the form

$\frac{8r^3}{(ry+q)^3} + \frac{4pr^2}{(ry+q)^2} - \frac{2qr}{ry+q} + r = 0$

$\Rightarrow 8r^3 - 4pr^2(ry+q) + 2qr(ry+q)^2 - r(ry+q)^3 = 0$

$r(ry+q)^3 - 2qr(ry+q)^2 + 4pr^2(ry+q) - 8r^3 = 0$