(7) of this eqn

The Constant term of this eqn

$= -8r^3 + 4pqr^2 - 2q^3r + q^3r$

The product of the roots of this eqn

$= -\dfrac{-8r^3 + 4pqr^2 - 2q^3r + q^3r}{r^4}$

$= \dfrac{1}{r^3}(8r^2 - 4pqr + q^3).$

(9) $\Sigma\alpha = -p$

$\Sigma\alpha\beta = q$

$\alpha\beta\gamma = -r$

We know $(a+b+c)^3 = a^3+b^3+c^3+3(b+c)(c+a)(a+b)$

$= a^3+b^3+c^3+3[(a+b+c)(ab+bc+ca) - abc]$

So $\Sigma\alpha^3\beta^3 = \alpha^3\beta^3 + \beta^3\gamma^3 + \gamma^3\alpha^3$

$= (\alpha\beta+\beta\gamma+\alpha\gamma)^3 - 3[(\alpha\beta+\beta\gamma+\alpha\gamma)(\alpha\beta^2\gamma+\alpha^2\beta\gamma+\alpha\beta\gamma^2) - (\alpha\beta\gamma)^2]$

$= q^3 - 3[q \times \alpha\beta\gamma(\alpha+\beta+\gamma) - r^2]$

$= q^3 - 3[q \times (-r)(-p) - r^2]$

$= q^3 - 3(pqr - r^2)$

$= q^3 - 3pqr + 3r^2$

(10) ~~Ex~~ If α, β, γ are the roots of the eqn then we have

$\Sigma\alpha = -p, \quad \Sigma\alpha\beta = q, \quad \alpha\beta\gamma = -r$

Now $\Sigma\dfrac{1}{\alpha} = \dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma} = \dfrac{\alpha\beta+\alpha\gamma+\beta\gamma}{\alpha\beta\gamma} = -$

$\Sigma\dfrac{1}{\alpha\beta} = \dfrac{1}{\alpha\beta} + \dfrac{1}{\alpha\gamma} + \dfrac{1}{\beta\gamma} = \dfrac{\alpha+\beta+\gamma}{\alpha\beta\gamma} =$

Part-C (8)

Cardan's Method Solution

Note

Let us Consider the cubic equation

$$a_0x^3+a_1x^2+a_2x+a_3=0 \quad \text{--- (1)}$$

If we want to diminish the roots of equ (1) by h, then we have to put

$$x=y+h \quad \text{--- (2)}.$$

Putting $x=y+h$ in (1) we get,

$$a_0(y+h)^3+a_1(y+h)^2+a_2(y+h)+a_3=0$$

$$\Rightarrow a_0\{y^3+3y^2h+3yh^2+h^3\}+a_1\{y^2+2yh+h^2\} + (a_2y+a_2h+a_3)=0$$

$$\Rightarrow a_0y^3+(3a_0h+a_1)y^2+(3h^2a_0+2ha_1+a_2)y + (a_0h^3+a_1h^2+a_2h+a_3)=0 \quad \text{--- (3)}$$

Since the 2nd term will be removed, So we must have

$$3a_0h+a_1=0$$

$$\Rightarrow h=-\frac{a_1}{3a_0}=-\frac{1}{3}\cdot\frac{a_1}{a_0}$$

Now,

$$(3) \Rightarrow a_0y^3+(3h^2a_0+2ha_1+a_2)y+(a_0h^3+a_1h^2+a_2h+a_3)=0 \quad \text{--- (4)}$$

If the roots of the equ^n (4) ~~are~~ be α, β and γ then the roots of the equation (1) will

(9)

be $\alpha+h, \beta+h, \gamma+h$

i.e $\alpha-\frac{a_1}{3a_0}, \beta-\frac{a_1}{3a_0}, \gamma-\frac{a_1}{3a_0}$.

Problems-A

Remove the 2nd term of the following equations —

1) $x^3-6x^2+2x+5=0$ Ans. $y^3-10y-7=0$
2) $2x^3-18x^2+3x+17=0$ Ans $2y^3-51y-92=0$
3) $x^3+6x^2+7x+2=0$ Ans. $y^3-5y+4=0$
4) $x^3-3x^2+12x+16=0$ Ans. $y^3+9y+26=0$
5) $6x^3-18x^2-19x-6=0$ $6y^3-37y-37=0$
6) $x^3-6x^2-6x-7=0$ Ans. $y^3-18y-35=0$

Problems-B

Solve the following equations by Cardan's Method.

- $\cancel{x^3-3x^2+12x+16=0}$
- $\cancel{x^3-9x^2-28=0}$
- $x^3-30x+133=0$
- $x^3-6x-9=0$
- $x^3-12x+65=0$
- $\cancel{x^3-6x^2-6x-7=0}$

B (7) $x^3-27x-54=0$

B (8) $x^3+6x+7=0$

B (9) $x^3-9x-28=0$

B (10) $x^3-18x-35=0$

(11) $\cancel{x^3-36x-204=0}$

(12) $x^3+30x-117=0$

(13) $x^3+24x+56=0$

(14) $x^3+15x-124=0$

(15) $x^3-45x-152=0$

(16) $x^3-9x+28=0$